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A scheme is proposed for preparing delocalised mesoscopic states of the motion of two or more atoms trapped at distantly-separated locations. Generation of entanglement is achieved using interactions in cavity quantum electrodynamics which facilitate motional quantum state transmission, via light, between separate nodes of a quantum network. Possible applications of the scheme are discussed.

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I. INTRODUCTION

The superposition principle and nonlocality are aspects of quantum mechanics that have teased and enticed the scientific and general communities alike since its development earlier this century. More specifically, conceptual difficulties arise when one tries to reconcile these features with the behaviour of the macroscopic, everyday world with which we are most familiar. These difficulties are exemplified by the classic Schrödinger Cat [1] and Einstein-Podolsky-Rosen (EPR) [2] paradoxes.

The boundary between the quantum and classical worlds remains a very active area of research, with, in particular, the atomic physics and quantum optics communities recently providing a number of significant experimental demonstrations. These include the preparation of mesoscopic superposition states – in particular, superpositions of small-amplitude coherent states – of a material oscillator (a trapped atom) [3] and of a cavity radiation field [4], and the generation of quantum-mechanically entangled internal states of pairs [5,6] and quadruplets [7] of atoms. These experiments open the door to detailed and highly-controlled investigations of quantum measurement, quantum decoherence, and nonlocality in mesoscopic systems and with massive particles.

Further along these lines, in this work we consider a combination of the technologies used in the above demonstrations, that is, single-atom trapping and cavity quantum electrodynamics (QED), to propose a scheme for the preparation of, arguably, a yet more exotic quantum state – a delocalised mesoscopic superposition state of two or more distantly-separated trapped atoms; that is, a state of the form

$$\frac{1}{\sqrt{N}} (|\alpha\rangle^1|0\rangle^2 \cdots |0\rangle^N + |0\rangle^1|\alpha\rangle^2 \cdots |\alpha\rangle^N + \cdots + |0\rangle^1|0\rangle^2 \cdots |\alpha\rangle^N), \quad (1)$$

where $|\alpha\rangle^j$ denotes a coherent state (in one dimension)

of the *motion of the atom* at the j -th site, and N is a normalisation factor. The term ‘mesoscopic’ is used because the coherent state $|\alpha\rangle^j$ is a ‘classical-like’ state, for which the mean number of quanta, $\bar{n} = |\alpha|^2$, may be relatively large (within the constraints of our scheme).

Before continuing, we make note of proposals that exist for the preparation of separated *light fields* in states of the form (1) with $N = 2$ (see, for example, [8,9]). Again though, it should be emphasised that here we are dealing with the states of *massive particles*.

II. QUANTUM STATE EXCHANGE BETWEEN MOTION AND LIGHT

Central to the proposed scheme is the ability to exchange quantum states between the motion of a trapped atom and a light field, allowing long-distance transmission of quantum information. As described in recent work [10], this can be done using a set of interactions in cavity QED, which we now summarise.

A. Trapped atom coupled to an optical cavity mode

The basic setup used was originally considered by Zeng and Lin [11]. This setup consists of a two-level atom (or ion) confined in a harmonic trap located inside an optical cavity. The atomic transition of frequency ω_0 is coupled to a single mode of the cavity field of frequency ω_{cav} and is also driven by an external (classical) laser field of frequency ω_A . The physical setup and excitation scheme are depicted in Fig. 1, with the relevant internal atomic levels being $|\uparrow\rangle$ and $|e\rangle$. The cavity is aligned along the x -axis, while the laser field [laser A in Fig. 1(b)] is incident from a direction in the y - z plane. The additional lasers B, C, and D and internal atomic levels $|\downarrow\rangle$ and $|f\rangle$ will be discussed later, but can be ignored for the moment.

The Hamiltonian describing the atom-cavity system takes the form (in a frame rotating at the laser frequency ω_A)

$$\begin{aligned} \hat{H} = & \sum_{j=x,y,z} \hbar\nu_j (\hat{b}_j^\dagger \hat{b}_j + 1/2) + \hbar\delta \hat{a}^\dagger \hat{a} + \hbar\Delta \hat{\sigma}_+ \hat{\sigma}_- \\ & + \hbar [\mathcal{E}_A(\hat{y}, \hat{z}, t) \hat{\sigma}_+ + \mathcal{E}_A^*(\hat{y}, \hat{z}, t) \hat{\sigma}_-] \\ & + \hbar g_0 \sin(k\hat{x}) (\hat{a}^\dagger \hat{\sigma}_- + \hat{\sigma}_+ \hat{a}) \\ & + \hat{a}^\dagger \hat{Y}_c + \hat{Y}_c^\dagger \hat{a} + \hat{\sigma}_+ \hat{Y}_a + \hat{Y}_a^\dagger \hat{\sigma}_-. \end{aligned} \quad (2)$$

Here, $\{\nu_x, \nu_y, \nu_z\}$ are the harmonic oscillation frequencies along the principal axes of the trap, \hat{b}_j and \hat{a} are annihila-

tion operators for the quantised atomic motion and cavity field, respectively, $\hat{\sigma}_- = |\uparrow\rangle\langle e|$ is the atomic lowering operator for the $|\uparrow\rangle \leftrightarrow |e\rangle$ transition, and $\delta = \omega_{\text{cav}} - \omega_A$ and $\Delta = \omega_0 - \omega_A$. The quantity $\mathcal{E}_A(\hat{y}, \hat{z}, t)$ is the (time-dependent) amplitude of laser field A. The single-photon atom-cavity dipole coupling strength is given by g_0 , while the sine function describes the standing wave structure of the cavity field (we assume that the centre of the trap is located at a *node* of the cavity field), with $k = 2\pi/\lambda$ the wavenumber of the field and $\hat{x} = [\hbar/(2m\nu_x)]^{1/2}(\hat{b}_x + \hat{b}_x^\dagger)$. Finally, the last line in (2) describes the coupling of the internal cavity field mode to the ‘reservoir’ of external field modes (with $\hat{\Upsilon}_c$ the ‘reservoir annihilation operator’), which produces damping of the cavity field, and the coupling of the atomic transition to (vacuum) radiation field modes other than the cavity mode, which gives rise to free-space spontaneous emission [12]. Note that we neglect any forms of motional decoherence associated with the trap itself.

In [10] a number of assumptions and approximations are made in order to simplify the model. In particular:

1. The detunings of the light fields from the atomic transition frequency are assumed to be very large [that is, $\Delta \gg |\mathcal{E}_A(t)|, g_0, \delta, \nu_j$], enabling atomic spontaneous emission to be neglected and the internal atomic dynamics to be adiabatically eliminated.
2. The size of the harmonic trap is assumed to be small compared to the optical wavelength (Lamb-Dicke regime), enabling the approximations $\sin(k\hat{x}) \simeq \eta_x(\hat{b}_x + \hat{b}_x^\dagger)$ and $\mathcal{E}_A(\hat{y}, \hat{z}, t) \simeq \mathcal{E}_A(t)e^{-i\phi_A}$, where η_x is the Lamb-Dicke parameter for the x -axis. Note that this assumption ultimately places a restriction on the mean excitation numbers $\{\bar{n}_j\}$ associated with the vibrational state of the atom, since the atomic wavepacket broadens with increasing \bar{n}_j . We will return to this point in the discussion when we consider the experimental feasibility of the scheme
3. The cavity and laser fields are tuned so that $\delta = \omega_{\text{cav}} - \omega_A = \nu_x$; that is, the fields are tuned to drive Raman transitions between neighbouring vibrational levels [see Fig. 1(b)].
4. The trap frequency ν_x and cavity field decay rate κ are assumed to satisfy $\nu_x \gg \kappa \gg |(g_0\eta_x/\Delta)\mathcal{E}_A(t)|$. The first inequality allows a rotating-wave approximation to be made with respect to the trap oscillation frequency, while the second inequality enables an adiabatic elimination of the cavity field mode.

Given these conditions one can show that the description of the motional mode dynamics in the x direction can be reduced to the simple quantum Langevin equation

$$\dot{\hat{b}}_x \simeq -[\Gamma(t) + i\nu_x]\hat{b}_x + e^{i\phi_A}\sqrt{2\Gamma(t)}e^{-i\nu_x t}\hat{a}_{\text{in}}(t), \quad (3)$$

where

$$\Gamma(t) = \frac{1}{\kappa} \left[\frac{g_0\eta_x\mathcal{E}_A(t)}{\Delta} \right]^2. \quad (4)$$

The operator $\hat{a}_{\text{in}}(t)$ obeys the commutation relation $[\hat{a}_{\text{in}}(t), \hat{a}_{\text{in}}^\dagger(t')] = \delta(t - t')$ and describes the quantum noise input to the cavity field (in a frame rotating at the cavity frequency) through the partially transmitting input/output mirror [see Fig. 1(a)].

B. Motional-state transfer between distant sites

The significance of Eq.(3) is that it amounts to a simple coupling of the motional mode to propagating light modes external to the cavity. More precisely, from the input-output theory of optical cavities [12,13], it can be shown that the cavity output field is given, under the present circumstances, by

$$\hat{a}_{\text{out}}(t) \simeq -\hat{a}_{\text{in}}(t) - \sqrt{2\Gamma(t)}e^{i\nu_x t}\hat{b}_x(t), \quad (5)$$

where we have set $\phi_A = 0$ for simplicity. Following work by Cirac *et al.* [14] (see also [13]) on the transmission of a qubit between two nodes of a quantum network, it is shown in [10] that if the output field from one of our atom-cavity configurations is incident on a second such atom-cavity configuration, with the coupling between systems being *unidirectional*, then with suitably tailored laser pulses $\mathcal{E}_{A1}(t)$ and $\mathcal{E}_{A2}(t)$ applied at the two sites one may realise the motional state transfer (assuming both atoms to be in the internal state $|\uparrow\rangle$)

$$|\phi\rangle_x^1 |0\rangle_x^2 \rightarrow |0\rangle_x^1 |\phi\rangle_x^2, \quad (6)$$

where $|\phi\rangle_x$ is an *arbitrary* quantum state describing the motion along the x -axis. Such a state transfer configuration is illustrated schematically in Fig. 2, while example pulse shapes, specified through the effective coupling rates of the motional modes to the external light fields, $\Gamma_1(t)$ and $\Gamma_2(t)$, are [10]

$$\Gamma_1(t) = \Gamma \frac{e^{\Gamma t}}{e^{\Gamma t} + e^{-\Gamma t}}, \quad \Gamma_2(t) = \Gamma_1(-t), \quad (7)$$

assuming the transfer starts at $t = -\infty$ and concludes at $t = +\infty$, with Γ a constant [equal to the maximum value of $\Gamma_{1,2}(t)$] [15]. Armed with this capability, we are able to distribute quantum states of a material oscillator, and generate entanglement, between macroscopically-separated locations, leading to some fascinating possibilities, one of which we now consider.

III. PREPARATION OF A DELOCALISED MOTIONAL COHERENT STATE

In addition to the atom-cavity coupling, our scheme for the preparation of a delocalised coherent state utilises

auxiliary lasers B, C, and D and internal atomic levels $|\downarrow\rangle$ and $|f\rangle$. Lasers B and C induce coherent Raman transitions between the internal states $|\downarrow\rangle$ and $|\uparrow\rangle$, and their frequency difference is chosen to be resonant with the transition frequency $\omega_{\uparrow} - \omega_{\downarrow}$, so that the motional state of the trapped atom is unaffected [3]. It is further assumed that if the atom is in the internal state $|\downarrow\rangle$ then it does not ‘see’ the cavity field or the coupling laser A. Hence, the motional state transfer described above only occurs when the atoms are in the internal state $|\uparrow\rangle$. Laser field D, when switched on, resonantly excites the transition $|\downarrow\rangle \leftrightarrow |f\rangle$; detection (absence) of fluorescence from this transition projects the system onto the internal state $|\downarrow\rangle$ ($|\uparrow\rangle$) [3,16]. Finally, while we focus on the (quasi-classical) coherent state $|\alpha\rangle_x$, it should be noted that the scheme outlined below will work for an arbitrary motional state $|\phi\rangle_x$.

A. Two atoms

Consider first the configuration shown in Fig. 2. Atom 1 is assumed to be prepared, by some means (for example, using additional laser or electric fields [3,16]), in a coherent state of its motion along the x -axis, while its internal state is prepared, via lasers B1 and C1, as the superposition $2^{-1/2}(|\uparrow\rangle^1 + |\downarrow\rangle^1)$. Atom 2 is prepared in its ground motional state $|0\rangle_x^2$ and in the internal state $|\uparrow\rangle^2$. Applying the state transfer laser pulses (through lasers A1 and A2 at sites 1 and 2, respectively) produces the transformation

$$\begin{aligned} & \frac{1}{\sqrt{2}}(|\uparrow\rangle^1 + |\downarrow\rangle^1) |\alpha\rangle_x^1 |\uparrow\rangle^2 |0\rangle_x^2 \\ & \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle^1 |0\rangle_x^1 |\alpha\rangle_x^2 + |\downarrow\rangle^1 |\alpha\rangle_x^1 |0\rangle_x^2) |\uparrow\rangle^2. \end{aligned} \quad (8)$$

Importantly, the part involving $|\downarrow\rangle^1$ does not change, as, again, from this state atom 1 does not couple to the laser field A1 or to the cavity field (so, cavity 1 is not excited and no light propagates to cavity 2, which also means that atom 2 remains in its ground motional state). Next, at site 1, lasers B1 and C1 are used to produce the internal state transformations

$$\begin{aligned} |\uparrow\rangle^1 & \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle^1 - |\downarrow\rangle^1), \\ |\downarrow\rangle^1 & \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle^1 + |\downarrow\rangle^1), \end{aligned} \quad (9)$$

leading to an overall system state

$$\begin{aligned} & \frac{1}{2} [|\uparrow\rangle^1 (|0\rangle_x^1 |\alpha\rangle_x^2 + |\alpha\rangle_x^1 |0\rangle_x^2) \\ & + |\downarrow\rangle^1 (-|0\rangle_x^1 |\alpha\rangle_x^2 + |\alpha\rangle_x^1 |0\rangle_x^2)] |\uparrow\rangle^2. \end{aligned} \quad (10)$$

Correlated with each internal state of atom 1 is thus a motional coherent state delocalised between atoms 1 and

2. If now laser D1 is applied to atom 1, then a null detection of fluorescence (occurring 50% of the time) will project the system into the state

$$\frac{1}{\sqrt{N}} (|0\rangle_x^1 |\alpha\rangle_x^2 + |\alpha\rangle_x^1 |0\rangle_x^2) |\uparrow\rangle^1 |\uparrow\rangle^2. \quad (11)$$

Note that detection of fluorescence, or more particularly the spontaneous scattering of photons associated with this fluorescence, would be expected to perturb the motional state in an uncontrollable and undesirable manner.

B. Three atoms

By adding another atom-cavity system to the cascade of Fig. 2, it is straightforward to extend the above scheme to prepare a motional coherent state delocalised between *three* atoms. Taking now the initial state of the system to be

$$\frac{1}{\sqrt{5}} (2|\uparrow\rangle^1 + |\downarrow\rangle^1) |\alpha\rangle_x^1 |\uparrow\rangle^2 |0\rangle_x^2 |\uparrow\rangle^3 |0\rangle_x^3, \quad (12)$$

applying the state transfer pulses (lasers A1 and A2) to atoms 1 and 2 transforms this state to

$$\frac{1}{\sqrt{5}} (2|\uparrow\rangle^1 |0\rangle_x^1 |\alpha\rangle_x^2 + |\downarrow\rangle^1 |\alpha\rangle_x^1 |0\rangle_x^2) |\uparrow\rangle^2 |\uparrow\rangle^3 |0\rangle_x^3. \quad (13)$$

A suitable pulse is then applied by lasers B2 and C2 to atom 2, causing the internal state transformation

$$|\uparrow\rangle^2 \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle^2 + |\downarrow\rangle^2). \quad (14)$$

State transfer pulses (lasers A2 and A3) are now applied to atoms 2 and 3, inducing the transformation

$$|\uparrow\rangle^2 |\alpha\rangle_x^2 |\uparrow\rangle^3 |0\rangle_x^3 \rightarrow |\uparrow\rangle^2 |0\rangle_x^2 |\uparrow\rangle^3 |\alpha\rangle_x^3, \quad (15)$$

and producing an overall state

$$\begin{aligned} & \frac{1}{\sqrt{10}} (2|\uparrow\rangle^1 |\uparrow\rangle^2 |0\rangle_x^1 |0\rangle_x^2 |\alpha\rangle_x^3 \\ & + 2|\uparrow\rangle^1 |\downarrow\rangle^2 |0\rangle_x^1 |\alpha\rangle_x^2 |0\rangle_x^3 \\ & + |\downarrow\rangle^1 |\uparrow\rangle^2 |\alpha\rangle_x^1 |0\rangle_x^2 |0\rangle_x^3 \\ & + |\downarrow\rangle^1 |\downarrow\rangle^2 |\alpha\rangle_x^1 |0\rangle_x^2 |0\rangle_x^3) |\uparrow\rangle^3. \end{aligned} \quad (16)$$

Lasers B1, C1 and B2, C2 are finally applied to atoms 1 and 2 to generate the internal state transformations

$$\begin{aligned} |\uparrow\rangle^j & \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle^j - |\downarrow\rangle^j), \\ |\downarrow\rangle^j & \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle^j + |\downarrow\rangle^j), \end{aligned} \quad (17)$$

with $j = 1, 2$, after which the overall state becomes

$$\begin{aligned}
& \frac{1}{\sqrt{10}} | \uparrow \rangle^1 | \uparrow \rangle^2 (| 0 \rangle_x^1 | 0 \rangle_x^2 | \alpha \rangle_x^3 + | 0 \rangle_x^1 | \alpha \rangle_x^2 | 0 \rangle_x^3 \\
& \quad + | \alpha \rangle_x^1 | 0 \rangle_x^2 | 0 \rangle_x^3) | \uparrow \rangle^3 \\
& + \frac{1}{\sqrt{10}} | \downarrow \rangle^1 | \downarrow \rangle^2 (| 0 \rangle_x^1 | 0 \rangle_x^2 | \alpha \rangle_x^3 - | 0 \rangle_x^1 | \alpha \rangle_x^2 | 0 \rangle_x^3) | \uparrow \rangle^3 \\
& + \frac{1}{\sqrt{10}} | \uparrow \rangle^1 | \downarrow \rangle^2 (- | 0 \rangle_x^1 | 0 \rangle_x^2 | \alpha \rangle_x^3 + | 0 \rangle_x^1 | \alpha \rangle_x^2 | 0 \rangle_x^3) | \uparrow \rangle^3 \\
& + \frac{1}{\sqrt{10}} | \downarrow \rangle^1 | \uparrow \rangle^2 (- | 0 \rangle_x^1 | 0 \rangle_x^2 | \alpha \rangle_x^3 - | 0 \rangle_x^1 | \alpha \rangle_x^2 | 0 \rangle_x^3 \\
& \quad + | \alpha \rangle_x^1 | 0 \rangle_x^2 | 0 \rangle_x^3) | \uparrow \rangle^3. \tag{18}
\end{aligned}$$

Internal state detection of atom 2 in the level $| \uparrow \rangle^2$ (via laser D2) projects the system onto a state where, once again, correlated with each internal state of atom 1 is a delocalised motional coherent state, only now the state is delocalised between three atoms. A further internal state projection onto $| \uparrow \rangle^1$ will of course disentangle internal and external degrees of freedom to yield the state

$$\begin{aligned}
& \frac{1}{\sqrt{N}} (| 0 \rangle_x^1 | 0 \rangle_x^2 | \alpha \rangle_x^3 + | 0 \rangle_x^1 | \alpha \rangle_x^2 | 0 \rangle_x^3 + | \alpha \rangle_x^1 | 0 \rangle_x^2 | 0 \rangle_x^3) \\
& \cdot | \uparrow \rangle^1 | \uparrow \rangle^2 | \uparrow \rangle^3. \tag{19}
\end{aligned}$$

Extensions of the scheme to more than three atoms follow straightforwardly, although the probability of obtaining the desired sequence of projective measurement results obviously decreases.

IV. OTHER POSSIBILITIES

The cases considered above clearly represent only a small subset of the state manipulations that are in principle possible with the trapped-atom-cavity configuration. Other combinations and sequences of internal state transformations and motional state transfers can produce a wide variety of entangled and delocalised quantum states. While we have not explored the possibilities exhaustively, two particular examples seem worthy of note.

A. GHZ states

Consider again the state produced in the transformation of Eq. (8), only now with an arbitrary motional state $|\phi\rangle_x^j$ replacing the coherent state:

$$\frac{1}{\sqrt{2}} (| \uparrow \rangle^1 | 0 \rangle_x^1 | \phi \rangle_x^2 + | \downarrow \rangle^1 | \phi \rangle_x^1 | 0 \rangle_x^2) | \uparrow \rangle^2. \tag{20}$$

This has the form of a Greenberger-Horne-Zeilinger state for three entangled degrees of freedom [17]. One degree of freedom is associated with the internal state of atom 1, while the other two degrees of freedom correspond to the (external) motional states of atoms 1 and 2 [18].

To physically separate the three degrees of freedom involved in the GHZ state, one could implement the following strategy. First, lasers B1 and C1 are applied to atom 1 to produce the internal state transformation $\{ | \uparrow \rangle^1 \rightarrow - | \downarrow \rangle^1, | \downarrow \rangle^1 \rightarrow | \uparrow \rangle^1 \}$. Then, the output channel from cavity 1 is *redirected* to a *third* atom-cavity system (so that atom 2 does not participate in the ensuing state transfer), and appropriate state transfer pulses from lasers A1 and A3 produce the transformation

$$\begin{aligned}
& \frac{1}{\sqrt{2}} (- | \downarrow \rangle^1 | 0 \rangle_x^1 | \phi \rangle_x^2 + | \uparrow \rangle^1 | \phi \rangle_x^1 | 0 \rangle_x^2) | \uparrow \rangle^2 | \uparrow \rangle^3 | 0 \rangle_x^3 \\
& \rightarrow \frac{1}{\sqrt{2}} (- | \downarrow \rangle^1 | \phi \rangle_x^2 | 0 \rangle_x^3 + | \uparrow \rangle^1 | 0 \rangle_x^2 | \phi \rangle_x^3) | 0 \rangle_x^1 | \uparrow \rangle^2 | \uparrow \rangle^3. \tag{21}
\end{aligned}$$

The entanglement is now between the internal state of atom 1 and the motional states of atoms 2 and 3, with the three atoms possibly separated by large distances. Note that entanglement of the three motional degrees of freedom could then be achieved by applying (via laser fields) an internal-state-dependent motional transformation to atom 1; for example, a transformation $\hat{U}_1(\phi)$ such that $\hat{U}_1(\phi) | \uparrow \rangle^1 | 0 \rangle_x^1 \rightarrow | \uparrow \rangle^1 | \phi \rangle_x^1$, while $\hat{U}_1(\phi) | \downarrow \rangle^1 | 0 \rangle_x^1 \rightarrow | \downarrow \rangle^1 | 0 \rangle_x^1$.

B. Hardy state

Consider the situation in which the initial motional state of atom 1 is prepared as the simple superposition of Fock states

$$| \phi \rangle_x^1 = a | 0 \rangle_x^1 + b | 1 \rangle_x^1, \quad (|a|^2 + |b|^2 = 1). \tag{22}$$

Following the procedure outlined by Eqs. (8–11) the (nonmaximally) entangled motional state that results is

$$\frac{1}{\sqrt{2(|a|^2 + 1)}} (2a | 0 \rangle_x^1 | 0 \rangle_x^2 + b | 0 \rangle_x^1 | 1 \rangle_x^2 + b | 1 \rangle_x^1 | 0 \rangle_x^2), \tag{23}$$

which is an example of what may be called a *Hardy state* [19–21]. Note that, for the purpose of preparing such a state, $| 1 \rangle_x^1$ could be replaced by any Fock state $| n \rangle_x^1$ with $n > 0$, or indeed by any (possibly mesoscopic) state orthogonal to $| 0 \rangle_x^1$.

V. DISCUSSION

A. Possible applications

The delocalised states prepared by the present scheme evidently offer a number of very interesting possibilities in a variety of different contexts, particularly given the exquisite control with which the states of trapped atoms can now be manipulated (see, for example, [16,22,23]),

and the flexibility and efficiency that is in principle possible with regards to measurements (see, for example, [22,24,25]). It is also worth emphasising again the ability to transfer the states of the motional modes onto propagating light fields with well-defined spatial and temporal properties [Eq.(5)]. The resulting delocalised light fields could of course be subjected to standard optical manipulations and measurements.

Some possible applications are to:

1. *Tests of quantum mechanics versus local realism:*
The GHZ and Hardy states just discussed allow for measurement outcomes from a *single* set of observations that are forbidden by theories based on local realism ('nonlocality without inequalities'). Entangled coherent states of the form $\mathcal{N}^{-1/2}(|0\rangle_x^1|\alpha\rangle_x^2 + |\alpha\rangle_x^1|0\rangle_x^2)$ also admit tests of local realism, but using Bell-type inequalities [8,26,27], while also offering new perspectives on complementarity [27]. In the context of tests of nonlocality, the fact that we are dealing with the states of *massive particles* is potentially also a very significant feature with regards to issues of causality [28].
2. *Quantum networks and error correction in quantum computation:*
The possibility of establishing entanglement and transferring quantum information between different nodes of a quantum network is naturally of great interest to the fields of quantum communication and quantum computing [29]. Indeed, quantum computer processors which are based on combinations of trapped-atom and cavity-QED methods and which communicate with other processors by optical means (via the cavity input-output coupling) have been proposed (see, for example, [30]) and are amongst the leading contenders for initial demonstrations of substantial quantum computations. States of the form $2^{-1/2}(|n\rangle^1|0\rangle^2 + |0\rangle^1|n\rangle^2)$ (with $n \geq 2$), where $|n\rangle^j$ is a Fock state, are also relevant to the idea of reversible measurements on a quantum system, or 'quantum jump inversion' [31], which is in turn of relevance to quantum error correction in quantum computing [32].
3. *Limits in interferometry:*
The state $2^{-1/2}(|n\rangle^1|0\rangle^2 + |0\rangle^1|n\rangle^2)$ is also known to allow the optimum $1/n$ phase sensitivity in a two-mode interferometer [22] and so is potentially of interest in the context of precision measurements.
4. *Tests of perception:*
If, for example, having prepared a state of the form $2^{-1/2}(|10\rangle_x^1|0\rangle_x^2 + |0\rangle_x^1|10\rangle_x^2)$, one switches on lasers A1 and A2 and separates the two cavity output fields in space, then the result will be a 10-photon light pulse delocalised between two distinct paths [33]. If these light pulses should be directed from distinct regions of space towards a single (human)

observer, then Ghirardi [34] has suggested the possibility of new investigations into the linearity (or nonlinearity) of quantum mechanics and the formation of definite perceptions.

B. Practical issues

The validity of, and constraints set by, the assumptions made in deriving the motion-to-light state exchange model that is central to the present work are discussed in more detail elsewhere [10,35]. Briefly, some of the more significant of these assumptions are as follows.

1. Lamb-Dicke assumption:

Taking into account the spread of the atomic wave function with increasing mean excitation number \bar{n}_x , the approximation $\sin(k\hat{x}) \simeq \eta_x(\hat{b}_x + \hat{b}_x^\dagger)$ can be associated with a condition of the form $\eta_x^2(1 + \bar{n}_x + 3\sigma_{\bar{n}_x}^2)/2 \ll 1$, where $\sigma_{\bar{n}_x}^2$ is the variance of the number state distribution [35]. If we consider, for example, a coherent state of mean excitation number $\bar{n}_x = |\alpha|^2 = 10$, this condition reduces to $\eta_x^2 \ll 0.1$.

2. Trap frequency and cavity linewidth:

Simulations of the atom-cavity configuration and the state transfer procedure show that the rotating-wave approximation (with respect to the trap frequency) requires that ν_x be at least five to ten times larger than the cavity field decay rate κ [10,35].

3. Atomic spontaneous emission:

Finite population in the atomic excited state leads to a finite degree of atomic spontaneous emission. For the effects of spontaneous emission on the atomic motional state to be negligible for the duration of the state transfer process (which occurs on a timescale of the order of Γ^{-1}), the condition $g_0^2/(\kappa\gamma) \gg 1$ should be satisfied [10]. This is simply the condition of strong coupling in cavity QED.

Lamb-Dicke parameters of the order of 0.1 or smaller, and trapping frequencies of several MHz to tens of MHz are now routinely achieved in ion-trapping experiments (see, for example, [16,23]). Optical dipole traps or magnetic traps should also be capable of confining neutral atoms in the Lamb-Dicke regime with frequencies in the MHz range (see, for example, [36,37]).

Also encouraging are the first generation of experiments in which single (neutral) atoms are trapped inside optical cavities in the regime of strong coupling cavity QED [36,38,39]. These experiments were not in a regime where the effective trapping frequency is larger than κ (although effective Lamb-Dicke parameters along the cavity axis were small), but given the aforementioned possibilities for trapping and given probable further improvements in mirror technology (that is, higher cavity

finesses), it seems reasonable to expect that future experiments trapping single atoms or ions inside optical cavities will be able to meet the various criteria of our scheme simultaneously.

An estimate for likely timescales involved in the present scheme can be given; if, for example, one assumes a trap frequency of $\nu_x/(2\pi) \simeq 5$ MHz and a cavity field decay rate of $\kappa/(2\pi) \simeq 500$ kHz, then an estimate for the rate of state transfer might be $\Gamma/(2\pi) \simeq 5 - 20$ kHz, or $\Gamma^{-1} \simeq 10 - 30 \mu\text{s}$. Note that the other basic operations involved in the scheme using the auxiliary lasers B,C, and D are routinely performed with very high efficiency in trapped-ion experiments, while the timescale for decoherence of motional states of single trapped ions is typically of the order of milliseconds or even longer [16,23,40]. This intrinsically slow decoherence rate of the motional state would allow for (relatively) long-lived delocalised quantum superpositions, which is an important and attractive feature of the proposal put forward in this work.

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FIG. 1. Schematic of proposed (a) setup and (b) excitation scheme for quantum state exchange between the motion of a trapped atom and a quantised cavity mode of the electromagnetic field. Note that all input and output to the atom-cavity system is through just one mirror; the other mirror is assumed to be perfect. The internal atomic structure shown in (b) is like that occurring in recent ion trap experiments. For simplicity, the vibrational level structure is shown only for the internal levels $|\uparrow\rangle$ and $|\downarrow\rangle$.

FIG. 2. Configuration for quantum transmission of the motional state of a trapped atom from site 1 to site 2. Lasers A1 and A2 are applied with time-dependent amplitudes of the forms shown, while Faraday isolators (F) facilitate a separation of input and output channels to and from the atom-cavity systems. Note that, during an ideal transfer, no light escapes from cavity 2 through its output channel.